Goods are packaged in a variety of containers. Sometimes the container is chosen because it packs well with similar containers.

Take a sheet of A4 card

Fold it to make a tall thin cuboid...

...or a short fat cuboid.

Both of these pack well. Which of them has the greater volume?

Find other ways you can fold a sheet of A4 to make the sleeve of a container which packs well.

Which container has the greatest volume?
Description

This topic investigates effective shapes for containers. It is a challenging and potentially mathematically wide-ranging activity.

Activity 1: Container sleeves

Container sleeves explores the most effective shape for the sleeve of a container – the shape that can hold the greatest volume for a given surface area. Introduce the activity with a discussion about possible shapes for containers which also considers which shapes pack well, such as prisms with a triangular, square or hexagonal cross section.

All of the sleeves in this investigation have the same surface area, that of an A4 sheet of thin card. (A4 measures 21cm by 29.8cm.) Ask your pupils, working in small groups, to fold A4 sheets of thin card to make a range of possible container shapes which pack well. This opens up opportunities to find volumes of prisms and therefore areas of a wide range of polygons, some regular, and many irregular. These polygons all need to tile. Encourage them to investigate the mathematical constraints this imposes.

Before they begin any calculations encourage discussion in groups to agree which of the two volumes will be larger for each polygon. For some polygons, for example, rectangles, the task of producing the two mathematically similar shapes will be demanding and a worthwhile exploration in itself. For each polygon, pupils then find and record the volumes that can be contained by their containers and compare their results to their expectations. Finding the cross-sectional areas for some of their containers may be challenging. They may be surprised that the shorter fatter containers have the greater volume.

The activity is wide ranging and you could choose, for example, to consider only regular polygons or use the activity as a vehicle to draw out that all triangles tile. Alternatively, you could make work on ratios a focus for the activity and restrict prisms to cuboids and systematically investigate the two possible volumes for cross-sectional areas with different ratios of sides, 1:1, 1:2, 1:3 and so on to find that the container with the square cross-sectional area is the most effective. If you leave the problem wide open you may need to encourage the pupils to focus down onto one of these investigations.

The mathematics

The activity requires pupils to calculate areas of polygons and to appreciate the link with volumes of prisms. It may also involve detailed exploration of similarity.